

# Flavor-Changing Processes in Extended Technicolor

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We analyze constraints on a class of extended technicolor (ETC) models from neutral flavor-changing processes induced by (dimension-six) four-fermion operators. The ETC gauge group is taken to commute with the standard-model gauge group. The models in the class are distinguished by how the left- and right-handed ( $L, R$ ) components of the quarks and charged leptons transform under the ETC group. We consider  $K^0 - \bar{K}^0$  and other pseudoscalar meson mixings, and conclude that they are adequately suppressed if the  $L$  and  $R$  components of the relevant quarks are assigned to the same (fundamental or conjugate-fundamental) representation of the ETC group. Models in which the  $L$  and  $R$  components of the down-type quarks are assigned to relatively conjugate representations, while they can lead to realistic CKM mixing and intra-family mass splittings, do not adequately suppress these mixing processes. We identify an approximate global symmetry that elucidates these behavioral differences and can be used to analyze other possible representation assignments. Flavor-changing decays, involving quarks and/or leptons, are adequately suppressed for any ETC-representation assignment of the  $L$  and  $R$  components of the quarks, as well as the leptons. We draw lessons for future ETC model building.

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## I. INTRODUCTION

It is possible that electroweak symmetry breaks via the formation of a bilinear condensate of fermions with a new strong gauge interaction, generically called technicolor (TC). To communicate this symmetry breaking to the standard model (technisinglet) fermions, one embeds technicolor in a larger, extended technicolor (ETC) theory [1]. To satisfy constraints from flavor-changing neutral-current (FCNC) processes, the ETC vector bosons that mediate generation-changing transitions must have large masses. To produce the hierarchy in the masses of the observed three generations (families) of fermions, the ETC masses arise from the sequential breaking of the ETC gauge symmetry on mass scales ranging from  $10^3$  TeV down to the TeV level. Precision measurements place tight constraints on these models, suggesting that there are a small number of new degrees of freedom at the TeV scale and that the technicolor theory has an approximately conformal (“walking”) behavior with large anomalous dimensions [2] - [4].

A class [5] - [8] of ultraviolet(UV)-complete ETC models takes the ETC dynamics to consist of a strongly interacting gauge theory whose gauge group commutes with the standard model (SM) gauge group. With the ETC representation assignments of the SM fermions depending on their assignments under the standard model, features such as intra-family mass splitting and CKM mixing can emerge. The ingredients to drive the ETC breaking are present. The models are distinguished by how the left ( $L$ )- and right ( $R$ )-handed quarks transform under the ETC group. This assignment must also be made for the charged leptons, but the choice is not critical for the considerations of this paper. The models include a mass-generation mechanism for neutrinos [6,8], although neutrino masses and mixing will not play an important role here.

Here we analyze the consequences of (dimension-6) four-fermion operators that occur in the effective theory at energies below  $\Lambda_{TC}$  in this class of ETC models, taking into account the multi-scale nature of the ETC gauge symmetry breaking along with mixing between ETC interaction eigenstates to form mass eigenstates. For a discussion of other phenomenologically relevant quantities, affected by dimension-5 operators, we refer the reader to [9,10].

In Section II, before specifying the details of the class of models of interest here, we present an effective field theory argument leading to the conclusion that ETC theories, even with walking, may generate phenomenologically unacceptable flavor-changing neutral current (FCNC) transitions, and describe a simple symmetry requirement for the underlying ETC dynamics such as to eliminate this problem. In Section III, we describe the structure of our class of UV-complete models. In the subsequent sections, we present estimates of the contributions of four-fermion operators produced by ETC to  $K^0 - \bar{K}^0$  mixing and other pseudoscalar mixing, as well as other processes. We show that when quarks of a given electric charge couple vectorially (in the fundamental or anti-fundamental representation) to the ETC gauge field, constraints from flavor-changing neutral current processes can be satisfied. We also consider FCNC constraints when the  $L$  and  $R$  quarks of a given charge are placed in relatively conjugate (fundamental and anti-fundamental) ETC representations. If this is done for the down-type quarks with the up-type quarks transforming vectorially, the model is ca-

pable of producing adequate intra-family fermion mass splittings and Cabibbo-Kobayashi-Maskawa (CKM) mixing. But this assignment does not suppress FCNC processes sufficiently. In Section VI, we summarize and draw conclusions for future model building, suggesting the use of other assignments.

## II. A GENERATIONAL SYMMETRY AND FOUR-FERMION OPERATORS.

ETC models, even in the presence of TC walking, can face a conflict between the requirement of generating large enough masses for the standard model fermions and the simultaneous requirement of not generating unacceptably large FCNC processes. In this section, we review why this is the case. In the next section, we show that some models in the class being considered can have the necessary symmetry structure to evade these arguments.

Consider the effective theory at an energy  $E > \Lambda_{TC}$  (the TC confinement scale), but below all the (larger) ETC scales. The fermion spectrum consists of all the SM fields and the technifermions. Having integrated out the ETC bosons, the resultant four-fermion operators (all of which preserve the full SM and TC gauge symmetries) can be classified in three groups: those involving only TC fields, which have no direct effect on the low-energy phenomenology, so that we will not discuss them further, those involving only ordinary fermions, and those that couple two technifermions and two ordinary fields. Each operator arises multiplied by the inverse square of an ETC scale and a dimensionless coefficient.

To construct the effective theory for  $E < \Lambda_{TC}$ , physics at the TC-scale is integrated out. The operators that couple two technifermions and two ordinary fermions produce, through the formation of TC-condensates, the dimension-3 bilinear fermion operators giving the ordinary fermion mass matrices. As an example, consider the mass of the down quark. It arises from the operator  $[\bar{d}_L \gamma_\mu D_L][\bar{D}_R \gamma^\mu d_R]$ , where  $D$  is a down-type techniquark, and can be estimated to be

$$m_d \simeq \kappa \eta \frac{\Lambda_{TC}^3}{\Lambda_1^2} , \quad (2.1)$$

where  $\Lambda_1$  is the highest ETC scale, associated with the first family,  $\kappa \sim O(10)$  is a numerical factor calculated in Ref. [8], and  $\eta$  is a factor incorporating walking, which can plausibly be of order the ratio of the lowest ETC scale to the TC scale ( $O(10)$ ), but is unlikely to be larger. A realistic value for the mass  $m_d$  can then be obtained naturally with the (large) value  $\Lambda_1 \sim 10^3$  TeV, and with  $\Lambda_{TC} \sim 300$  GeV as dictated by the scale of electroweak symmetry breaking.

The operators containing only ordinary (technisinglet) fermions remain in the lower energy theory and is responsible for the FCNC transitions of concern in this paper. Consider for example the  $K^0 \leftrightarrow \bar{K}^0$  mixing amplitude, generated by four-fermion operators of the form  $[\bar{d}_{\chi'} \gamma_\mu s_\chi][\bar{d}_{\chi'} \gamma^\mu s_\chi]$ , where  $\chi\chi' = LL, LR, RR$ . The standard model produces an operator of this type with  $\chi\chi' = LL$  and a coefficient that can fit experiment. In ETC models there are typically additional contributions to these operators. For example, in certain ETC models there is a contribution to a  $K^0 - \bar{K}^0$  operator (of LR type) at the scale  $\Lambda_1$ . Taking

the coefficient of this operator to be  $b/\Lambda_1^2$ , where  $b$  is a dimensionless number, the requirement of having a sufficiently small contribution to  $\Delta m_K$  implies

$$\frac{b}{\Lambda_1^2} \lesssim \frac{\text{few} \times 10^{-14}}{\text{GeV}^2} . \quad (2.2)$$

With  $b = \mathcal{O}(1)$ , this would require  $\Lambda_1 \gtrsim 10^4$  TeV, an order of magnitude larger than the (already large) value required to give a realistic down-quark mass.

There is a natural way out of this problem, incorporated into some of the models considered here. The four-fermion operators relevant for the down-quark mass and for  $K^0 \leftrightarrow \bar{K}^0$  mixing, although generated at the same scales, can have different symmetry properties with respect to the underlying ETC gauge theory. This in turn can lead to  $b \ll 1$  while the corresponding coefficient in the mass-generating four-fermion operator is  $\mathcal{O}(1)$ . This is expected since the first operator involves four fields carrying ordinary flavor quantum numbers, while the second involves just two.

For all the models we consider, the theory at energies below  $\Lambda_{TC}$  contains an approximate global generational  $U(1)^3$  symmetry, with one  $U(1)$  factor associated with each family of SM-fermions, and with each (chiral) member of a family carrying its own  $U(1)$  charge. This symmetry is a remnant of the underlying ETC gauge symmetry, and the charge assignments are determined by how each fermion transforms under the ETC gauge group. For the suppression (2.2) to be present, the left-handed and right-handed down-type quarks of the first and second generation must be in ETC representations such that the operator  $[\bar{d}_L \gamma_\mu s_L][\bar{d}_R \gamma^\mu s_R]$  violates the global  $U(1)^3$  symmetry. (Clearly, the corresponding operators with all four fields of the same chirality violate the symmetry.)

If this is the case, then the  $LR$  operator could not be generated by ETC exchange if the global generational  $U(1)^3$  symmetry were exact. But in the spontaneous breaking of the ETC gauge group, this symmetry is broken, by mixing terms between different ETC gauge bosons. However, the mixing involves ratios of the hierarchical scales of ETC symmetry breaking, and is therefore small, strongly suppressing the contributions to the  $LR$  operator.

### III. ETC MODELS

We take the ETC gauge group  $G_{ETC}$  to commute with the SM group  $G_{SM} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ . The ETC group gauges the three generations of technisinglet fermions and connects them with the technicolored fermions. We use  $G_{ETC} = \text{SU}(N_{ETC})$ , with the TC group  $\text{SU}(N_{TC}) \subset G_{ETC}$ . For several reasons (see below), we choose  $N_{TC} = 2$ , and hence  $G_{ETC} = \text{SU}(5)_{ETC}$ . The ETC gauge symmetry is chiral, so that when it becomes strong, sequential breaking occurs naturally. This breaking also involves one additional strongly coupled gauge interaction. The breaking of the  $\text{SU}(5)_{ETC}$  to  $\text{SU}(2)_{TC}$  is driven by the condensation of SM-singlet fermions which are part of the models; while there is some freedom in the actual choice of

these singlets, their presence is always mandatory in order to cancel  $SU(5)_{ETC}$  anomalies. The SM-singlet fermions that condense acquire large masses, and hence decouple from the low-energy effective theory.

The ETC symmetry breaking takes place in stages, so that  $SU(5)_{ETC} \rightarrow SU(4)_{ETC}$  at a scale  $\Lambda_1$ , with the first-generation SM fermions separating from the others; then  $SU(4)_{ETC} \rightarrow SU(3)_{ETC}$  at a lower scale  $\Lambda_2$  and  $SU(3)_{ETC} \rightarrow SU(2)_{TC}$  at a still lower scale  $\Lambda_3$ , with the second- and third-generation fermions separating in the same way, leaving the technifermions. As  $SU(N)_{ETC}$  breaks to  $SU(N-1)_{ETC}$  at the scale  $\Lambda_j$ , the  $2N-1$  ETC gauge bosons in the coset  $SU(N)/SU(N-1)$  gain masses

$$M_j \simeq \frac{g_{ETC} a \Lambda_j}{4} \quad (3.1)$$

where  $a \simeq O(1)$ . Since  $g_{ETC}^2/(4\pi) \simeq O(1)$ , it follows that  $M_j \simeq \Lambda_j$ . Following our earlier work [6,8–10], we take these scales for definiteness to be

$$\Lambda_1 = 10^3 \text{ TeV}, \quad \Lambda_2 = 10^2 \text{ TeV}, \quad \Lambda_3 = 4 \text{ TeV}. \quad (3.2)$$

At the scale  $\Lambda_{TC}$ , technifermion condensates break the electroweak symmetry.

The choice  $N_{TC} = 2$  has the advantages that it (a) minimizes the TC contributions to the electroweak  $S$  parameter, (b) with a SM family of technifermions,  $Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L$ ,  $L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L$ ,  $U_R$ ,  $D_R$ ,  $N_R$ ,  $E_R$  transforming according to the fundamental representation of  $SU(2)_{TC}$ , can yield an approximate infrared fixed point and the associated walking behavior [2,3] and (c) makes possible a mechanism to account for light neutrinos without any super-heavy mass scale [6,7].

Each of the above technifermions together with a set of ordinary fermions with the same SM quantum numbers is placed in a representation of  $SU(5)_{ETC}$ . In each case, the charge assignments of the components under the approximate global  $U(1)^3$  described in Section II depend on the  $SU(5)_{ETC}$  representations. For a fermion  $(\psi_\chi)_{j_1 \dots j_n}^{i_1 \dots i_m}$  of chirality  $\chi = L, R$  transforming according to a general representation of  $SU(5)_{ETC}$ , the charge  $Q_k$  of a given component under the  $k$ 'th  $U(1)$  of the  $U(1)^3$ , for  $k = 1, 2, 3$ , is

$$Q_k = \sum_{p=1}^m \delta_{k,i_p} - \sum_{q=1}^n \delta_{k,j_q}. \quad (3.3)$$

Consider, for example, the left-handed quark-techniquark electroweak doublet. If it is assigned to the fundamental (anti-fundamental) ETC representation, then the  $U(1)^3$ -charge assignment of its first-family members is  $(\pm 1, 0, 0)$ , etc. As we will see, more general representational assignments may be necessary to produce fully realistic models.

In previous work we have analyzed two types of ETC models in the general class [5] - [10]. In one (denoted CSM in Ref. [8]),  $L$  quarks and  $R$  up-type quarks are assigned to the fundamental representation of  $SU(5)_{ETC}$ , while the  $R$  down-type quarks transform according to the conjugate fundamental representation [11]. These models exhibit charged-current CKM flavor mixing, intra-generational mass splittings without excessive contributions to  $\rho - 1$  where  $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)$ , as well as the natural appearance of

CP-violating phases. However, they give rise to the operator  $[\bar{d}_L \gamma_\mu s_L][\bar{d}_R \gamma^\mu s_R]$  without violating the  $U(1)^3$  global symmetry, and therefore give an excessive ETC contribution to the  $K^0 \leftrightarrow \bar{K}^0$  mixing amplitude.

In another type of model (denoted VSM in Ref. [8]), the  $L$  and  $R$  components of all the quarks and techniquarks transform according to the fundamental representation of the ETC group [5]- [7]. Without additional ingredients these models cannot lead to realistic CKM mixing and intra-family mass splittings. The vectorial structure of these models does, however, naturally lead to adequate suppression of flavor-changing neutral current processes (for example, the operator  $[\bar{d}_L \gamma_\mu s_L][\bar{d}_R \gamma^\mu s_R]$  in this case violates the  $U(1)^3$ ). Although the vectorial structure was typical of early ETC model building, the natural FCNC suppression seems not to have been noticed.

Clearly neither type of model is fully realistic, and it will be important to extend the class, exploring the assignment of the fermion fields to other representations of the ETC group, and possibly including additional interactions at energies not far above  $\Lambda_1$ . Such modifications are also needed to eliminate one other problem with the class of models. They all have a small number of unacceptable Nambu-Goldstone bosons arising from spontaneously broken  $U(1)$  global symmetries. These must be removed or given sufficiently large masses to have escaped detection. In this paper, as we consider each physical process, we take, for simplicity, the  $L$  and  $R$  components of the relevant quarks (quarks of a given charge) to transform according to either the same (fundamental or anti-fundamental) ETC representations or to relatively conjugate (fundamental and anti-fundamental) representations. The latter choice, as indicated above, will give excessive contributions to  $K^0 \leftrightarrow \bar{K}^0$  and other mixing. We comment on other possible ETC representation assignments in the summary Section VI.

### A. ETC Gauge Bosons

Each SM quark or charged lepton of a given chirality  $\chi = L, R$  is embedded in a 5 or  $\bar{5}$  representation of  $SU(5)_{ETC}$  so that the components with indices  $i = 1, 2, 3$  correspond to the three generations, and those with indices  $i = 4, 5$  are the technifermions. For a fermion  $f_\chi$  transforming as a 5 of  $SU(5)_{ETC}$ , the basic coupling to the ETC gauge bosons is

$$\mathcal{L} = g_{ETC} \bar{f}_{j,\chi} (T_a)_k^j (V_a)^\lambda \gamma_\lambda f_\chi^k \quad (3.4)$$

where the  $T_a$ ,  $a = 1, \dots, 24$  are the generators of the Lie algebra of  $SU(5)_{ETC}$  and the  $V_a$  are the corresponding ETC gauge fields. Similarly, if  $f_\chi$  transforms as a  $\bar{5}$ , this coupling is  $g_{ETC} \bar{f}_\chi^j (T_a)_j^k (V_a)^\lambda \gamma_\lambda f_{k\chi}$ . For nondiagonal transitions,  $j \neq k$ , it is convenient to use the fields  $V_k^j = \sum_a V_{a,\lambda} (T_a)_k^j$ , whose absorption by  $f_\chi^k$  yields  $f_\chi^j$ , with coupling  $g_{ETC}/\sqrt{2}$ , analogous to the  $W^\pm$  in  $SU(2)_L$ . We take the diagonal (Cartan) generators to be  $T_{24} \equiv T_{d1} = (2\sqrt{10})^{-1} \text{diag}(-4, 1, 1, 1, 1)$ ,  $T_{15} \equiv T_{d2} = (2\sqrt{6})^{-1} \text{diag}(0, -3, 1, 1, 1)$ ,  $T_8 \equiv T_{d3} = (2\sqrt{3})^{-1} \text{diag}(0, 0, -2, 1, 1)$ , and  $T_3 = (1/2) \text{diag}(0, 0, 0, -1, 1)$ . The ETC gauge bosons that couple to these diagonal generators  $T_{dj}$  are denoted  $V_{dj}$ .

When  $SU(5)_{ETC}$  breaks to  $SU(4)_{ETC}$ , the nine ETC gauge bosons in the coset  $SU(5)_{ETC}/SU(4)_{ETC}$ , namely,  $V_j^1$ ,  $(V_j^1)^\dagger = V_1^j$ ,  $j = 2, 3, 4, 5$ , and  $V_{d1}$ , gain masses  $M_1 \simeq \Lambda_1$ . Similarly, when  $SU(4)_{ETC}$  breaks to  $SU(3)_{ETC}$ , the seven ETC gauge bosons  $V_j^2$  and  $(V_j^2)^\dagger = V_2^j$ ,  $j = 3, 4, 5$ , together with  $V_{d2}$ , gain masses  $\simeq \Lambda_2$ . Finally, when  $SU(3)_{ETC}$  breaks to  $SU(2)_{TC}$ , the five ETC gauge bosons  $V_j^3$ ,  $(V_j^3)^\dagger = V_3^j$ ,  $j = 4, 5$ , together with  $V_{d3}$ , gain masses  $\simeq \Lambda_3$ . The SM-singlet fermions responsible for this breaking also, through quantum loops, lead to mixing among these gauge bosons, so that they are not exact mass eigenstates. The mixing is small, being suppressed by ratios of the hierarchical ETC scales.

These mixing terms among ETC gauge bosons are the source of breaking of the global generational  $U(1)^3$  symmetry introduced in Section II. In the absence of any such mixing, this symmetry would remain unbroken, and would forbid many four-fermion operators in the effective theory for  $E < \Lambda_{TC}$ . The smallness of the ETC mixing, together with the largeness of the ETC scales, will be crucial for the suppression of FCNC processes.

A particular type of ETC mixing will be focused on in this paper. This is the mixing among the ETC gauge bosons  $V_t^i$  that transform as doublets under  $SU(2)_{TC}$  (with ETC index  $t \in \{4, 5\}$ ) and triplets under the generational  $SU(3)$  (with ETC index  $i \in \{1, 2, 3\}$ ,  $j \neq k$ ). This can be of the form  $V_t^i \leftrightarrow V_t^j$ , producing the off-diagonal elements of the quark mass matrices when the  $L$  and  $R$  components transform according to the same representation. We note that the diagonal elements do not require mixing since  $f_L^i$  and  $f_R^i$  have the same charge under the  $U(1)^3$  generational symmetry introduced in Section II. Thus the off-diagonal elements are suppressed relative to the diagonal elements. The ETC mixing can also be of the form  $V_4^i \leftrightarrow V_5^j$  when the  $L$  and  $R$  components transform according to relatively conjugate ETC-representations. In this case,  $f_L^i$  has a different  $U(1)^3$ -charge than  $f_{jR}$  for all  $(i, j)$ , and therefore all the elements of the mass matrix are suppressed.

## B. Quark Masses

The effective theory describing the physics at energies  $E < \Lambda_{TC}$ , obtained by integrating out the ETC and TC gauge bosons and all the heavy fermions, contains the mass matrix of SM quarks or charged leptons, given by

$$\mathcal{L}_m = -\bar{f}_{j,L} M_{jk}^{(f)} f_{k,R} + h.c., \quad (3.5)$$

where  $f$  denotes up-type and down-type quarks, as well as charged leptons, and the indices  $j, k \in \{1, 2, 3\}$  are generation indices, all written as subscripts here. The structure of  $M^{(f)}$  depends on the type of ETC model that generates it. If the  $L$  and  $R$  components transform according to the same representation of the ETC group, then the diagonal elements of  $M^{(f)}$  do not require any ETC gauge boson mixing (being invariant under  $U(1)^3$ ), while the off-diagonal elements do require mixing. If the  $L$  and  $R$  components transform according to relatively conjugate representations, then ETC mixing is required for all the elements of  $M^{(f)}$ .

An arbitrary mass matrix  $M^{(f)}$  can be brought to real, positive diagonal form by the bi-unitary transformation

$$U_L^{(f)} M^{(f)} U_R^{(f)\dagger} = M_{diag}^{(f)} . \quad (3.6)$$

Hence, the interaction eigenstates  $f$  of the quarks are mapped to mass eigenstates  $q$  via

$$f_\chi = U_\chi^{(f)\dagger} q_\chi \quad (3.7)$$

for  $\chi = L, R$ . Writing out the vectors  $q_\chi$  explicitly for the up and down quarks, we have  $u_\chi = (u, c, t, U^4, U^5)_\chi$ ,  $d_\chi = (d, s, b, D^4, D^5)_\chi$ , and, for the leptons,  $(e, \mu, \tau, E^4, E^5)_\chi$ . Here we use the notation  $u$  and  $d$  to refer to the respective charge  $Q = 2/3$  and  $Q = -1/3$  five-dimensional representations of  $SU(5)_{ETC}$  and to the individual  $u$  and  $d$  quarks; the specific meaning will be clear from context. The observed CKM quark mixing matrix  $V$  that enters in the charged weak current is then given by

$$V = U_L^{(u)} U_L^{(d)\dagger} . \quad (3.8)$$

For the parametrization of the matrices  $U_\chi^{(f)}$ , we recall that a general  $N \times N$  unitary matrix  $U$  (where here  $N = 3$  generations) can be written as  $U = e^{i\phi} \mathcal{U}$  where  $\mathcal{U} \in SU(N)$ . The matrix  $U$  depends on  $N^2$  real parameters, of which  $N(N-1)/2 = 3$  are rotation angles, and the remaining  $N(N+1)/2 = 6$  are complex phases. Thus each of the matrices  $U_\chi^{(f)}$ ,  $\chi = L, R$ , depends on three angles  $\theta_{mn}^{(f)\chi}$ ,  $mn = 12, 13, 23$ , and six (independent) phases. Some of these phases can be removed by rephasings of quark fields, as we discuss further below. We parametrize the transformation matrices  $U_\chi^{(f)}$ ,  $\chi = L, R$ , as

$$U_\chi^{(f)} = e^{i\phi_{(f)\chi}} P_\alpha^{(f)\chi} U_{0\chi}^{(f)} P_\beta^{(f)\chi} \quad (3.9)$$

where

$$P_\alpha^{(f)\chi} = \text{diag}(e^{i\alpha_1^{(f)\chi}}, e^{i\alpha_2^{(f)\chi}}, e^{i\alpha_3^{(f)\chi}}) , \quad (3.10)$$

$$P_\beta^{(f)\chi} = \text{diag}(e^{i\beta_2^{(f)\chi}}, e^{i\beta_2^{(f)\chi}}, e^{i\beta_3^{(f)\chi}}) , \quad (3.11)$$

with  $\alpha_3^{(f)\chi} = -\alpha_1^{(f)\chi} - \alpha_2^{(f)\chi}$ ,  $\beta_3^{(f)\chi} = -\beta_1^{(f)\chi} - \beta_2^{(f)\chi}$ , and the matrix  $U_0^{(f)}$  follows the ordering conventions of [12],

$$U_{0\chi}^{(f)} = R_{23}(\theta_{23}^{(f)\chi}) P_\delta^{(f)\chi} * R_{13}(\theta_{13}^{(f)\chi}) P_\delta^{(f)\chi} R_{12}(\theta_{12}^{(f)\chi}) , \quad (3.12)$$

where  $R_{mn}(\theta_{mn}^{(f)\chi})$  is the rotation in the  $mn$  subsector, and

$$P_\delta^{(f)\chi} = \text{diag}(e^{i\delta^{(f)\chi}}, 1, 1) . \quad (3.13)$$

It is a convention [12] how one chooses to insert a phase like  $\delta$  among the rotations  $R_{12}$ ,  $R_{13}$ , and  $R_{23}$ , but physical quantities depend only on expressions that are independent of such conventions. Note that we have



not tried to remove a maximal number of phases to put the resultant quark mixing matrix  $V$  in its canonical form. When dealing with CP-violating quantities, we will therefore write explicitly rephasing-invariant expressions.

In the special case when the  $L$  and  $R$  components of the fields  $f$  transform in the same ETC representation, the vectorial nature of the ETC interactions responsible for generating their mass matrix implies

$$U_L^{(f)} = e^{-i\phi_{(f)R}} U_R^{(f)} \equiv U^{(f)}, \quad (3.14)$$

where  $U^{(f)}$  is unimodular. In this case,  $M^{(f)}$  is hermitian up to the phase factor  $e^{-i\phi_{(f)R}}$ . Consequently,  $\phi_{(f)L} = 0$ ,  $U_0^{(f)L} = U_0^{(f)R} \equiv U_0^{(f)}$ ,  $P_\alpha^{(f)L} = P_\alpha^{(f)R} \equiv P_\alpha^{(f)}$ , etc. and thus  $\theta_{mn}^{(f)L} = \theta_{mn}^{(f)R} \equiv \theta_{mn}^{(f)}$ ,  $\delta^{(f)L} = \delta^{(f)R} \equiv \delta^{(f)}$ ,  $\alpha_j^{(f)L} = \alpha_j^{(f)R} \equiv \alpha_j^{(f)}$ , and  $\beta_j^{(f)L} = \beta_j^{(f)R} \equiv \beta_j^{(f)}$ .

Since we will analyze CP-violation in neutral  $K$  and  $B_d$  mixing, a remark on the strong CP problem is in order. This is the problem of why  $|\bar{\theta}| \lesssim 10^{-10}$ , where  $\bar{\theta} = \theta - [\arg(\det(M^{(u)})) + \arg(\det(M^{(d)}))]$ , with  $\theta$  appearing via the topological term  $\theta g_s^2 (32\pi^2)^{-1} G_{a\mu\nu} \tilde{G}_a^{\mu\nu}$ . Whether a resolution of the strong CP problem will emerge in the class of models considered here is not yet clear [13]. However, whatever the resolution of the strong CP problem turns out to be,  $\bar{\theta}$  involves only the flavor-independent  $\phi_{(f)\chi}$  phases in the  $U_\chi^{(f)}$ ,  $\chi = L, R$ . By contrast, the CP-violating quantities considered here and in Ref. [10] depend on the other, generation-dependent phases in the  $U_\chi^{(f)}$ . Even in a theory that provides for a solution of the strong CP problem, one must analyze the effects of these flavor-dependent phases, as we do here. Aside from assuming that  $|\bar{\theta}|$  is sufficiently small, we will not make any special assumptions concerning the sizes of the CP-violating phases that enter in the  $U_\chi^{(f)}$ . This is in accord with the fact that the intrinsic CP violating phase  $\delta$  in the CKM matrix, is not small [14].

### C. Dimension-6 Four-Fermion Operators

Integrating out the ETC-scale physics produces not only the quark-mass operators, but also operators of higher dimension. In previous papers, we discussed the impact of the (dimension-5) dipole operators [9,10]. Here we are interested in the (dimension-6) four-fermion operators describing flavor-changing neutral-current processes. Since at each scale of ETC breaking, the ETC interactions are strong, the estimate of the four-fermion operators must be done to all orders in this coupling.

The four-fermion operators of interest receive two types of contributions from the exchange of heavy ETC gauge bosons, with ETC mixing and without. If the fermion assignments to the ETC gauge group are such that a four-fermion operator preserves the  $U(1)^3$  global symmetry, then its coefficient is suppressed only by the mass scale of the gauge boson exchanged; no ETC mixing is required. (For the processes considered in this paper, this scale will be  $\Lambda_1$ , with the exception of  $B_s - \bar{B}_s$  mixing, for which it will be  $\Lambda_2$ .) This will be problematic for certain neutral-pseudoscalar mixing processes.

If a four-fermion operator does not respect the  $U(1)^3$  global symmetry, its generation requires ETC mixing (which violates the global symmetry). This mixing can take place, for example, among the ETC

gauge bosons transforming as  $SU(2)_{TC}$ -doublets. It enters the four-fermion operators through the unitary matrices that diagonalize the fermion mass matrices. The coefficient of the resultant four-fermion operators is then suppressed not only by the masses of the exchanged ETC gauge bosons, but also by (small) mixing angles coming from Eqs. (3.7)-(3.12).

The ETC mixing and associated breaking of the global  $U(1)^3$  generational symmetry takes place also among the ETC gauge bosons transforming as TC- singlets. This mixing is not directly responsible for the generation of the SM-fermion mass matrices, but appears among the ETC gauge bosons exchanged between the SM-fermions. These mixings, too, are proportional to ratios of ETC scales, and hence suppressed, so that the contribution to four-fermion operators is small. There will be relations between this type of mixing and that among the  $SU(2)_{TC}$ -doublet gauge bosons. But we expect its effect on four-fermion operators to be at most comparable to that due to the doublet mixing, and therefore we will not consider it further in this paper.

To estimate the contribution to the four-fermion operators arising from the off-diagonal quark mass matrices, it will be helpful to re-express the ETC gauge couplings in terms of quark mass eigenstates. The full  $5 \times 5$  matrix  $\sum_{a=1}^{24} T_a V_a^\lambda$  that enters this coupling, may be restricted to its  $3 \times 3$  submatrix involving only ordinary quark indices. Suppressing the Lorentz index  $\lambda$ , we thus define

$$\mathcal{V} = \begin{pmatrix} -\frac{2V_{d1}}{\sqrt{10}} & \frac{V_2^1}{\sqrt{2}} & \frac{V_3^1}{\sqrt{2}} \\ \frac{V_1^2}{\sqrt{2}} & \frac{V_{d1}}{2\sqrt{10}} - \frac{3V_{d2}}{2\sqrt{6}} & \frac{V_3^2}{\sqrt{2}} \\ \frac{V_1^3}{\sqrt{2}} & \frac{V_2^3}{\sqrt{2}} & \frac{V_{d1}}{2\sqrt{10}} + \frac{V_{d2}}{2\sqrt{6}} - \frac{V_{d3}}{\sqrt{3}} \end{pmatrix} \quad (3.15)$$

If both the  $L$  and  $R$  components are in the fundamental representation, the coupling is vectorial and given by

$$\mathcal{L}_{int} = g_{ETC} \sum_{f,j,k} \bar{f}_j \gamma_\lambda (\mathcal{V}^\lambda)_k^j f^k \equiv g_{ETC} \sum_{q,j,k} \bar{q}_j \gamma_\lambda (A^\lambda)_k^j q^k, \quad (3.16)$$

with

$$A^\lambda \equiv U^{(f)} \mathcal{V}^\lambda U^{(f)-1}, \quad (3.17)$$

where we have used Eq. (3.14), and we recall from Eq. (3.7) that the  $f^j$  and  $q^k$  are the ETC interaction and mass eigenstates of the quarks of a given charge. We can simplify this expression by absorbing the  $P_\alpha^{(f)}$  in the  $q$  fields so that for these ETC models with vectorial SM fermion representations,

$$A^\lambda = U_0^{(f)} P_\beta^{(f)} \mathcal{V}^\lambda P_\beta^{(f)*} U_0^{(f)\dagger}, \quad (3.18)$$

which does not depend on  $P_\alpha^{(f)}$ . The above rephasing of the  $q$  fields leaves the diagonalized fermion mass matrix invariant.

If the  $L$  and  $R$  components of the quarks transform according to conjugate representations, then the ETC couplings are chiral and given by

$$\mathcal{L}_{int,L} = g_{ETC} \sum_{f,j,k} \bar{f}_{j,L} \gamma_\lambda (\mathcal{V}^\lambda)_k^j f_L^k \equiv g_{ETC} \sum_{f,j,k,\chi} \bar{q}_{j,L} \gamma_\lambda (A_L^{(f)\lambda})_k^j q_L^k, \quad (3.19)$$

and

$$\mathcal{L}_{int,R} = g_{ETC} \sum_{f,j,k} \bar{f}_R^j \gamma_\lambda (\mathcal{V}^\lambda)_j^k f_{k,R} \equiv g_{ETC} \sum_{f,j,k,\chi} \bar{q}_R^j \gamma_\lambda (A_R^{(f)\lambda})_j^k q_{k,R}, \quad (3.20)$$

where

$$A_\chi^{(f)\lambda} = U_\chi^{(f)} \mathcal{V}^\lambda [U_\chi^{(f)}]^{-1}. \quad (3.21)$$

#### IV. NEUTRAL PSEUDOSCALAR MESON MIXING

Owing to the transitions  $M^0 \leftrightarrow \bar{M}^0$ , where  $M^0 = K^0, B_d, B_s$ , or  $D^0$ , the mass eigenstates of these neutral non-self-conjugate mesons involve linear combinations of  $|M^0\rangle$  and  $|\bar{M}^0\rangle$ . The time evolution of the  $M^0, \bar{M}^0$  system is governed by  $M - i\Gamma/2$ , where  $M$  and  $\Gamma$  are  $2 \times 2$  hermitian matrices in the basis  $(M^0, \bar{M}^0)$ . The resultant physical mass eigenstates have different masses; we denote these as  $M_h^0$  and  $M_\ell^0$ , with mass difference  $\Delta m_M = m_{M_h^0} - m_{M_\ell^0}$ . For the kaon system,  $K_L = K_h$  and  $K_S = K_\ell$  are the long- and short-lived eigenstates, and  $\Delta m_K = 2\text{Re}(M_{12})$ . For the  $B$  and  $D$  mesons,  $M_h$  and  $M_\ell$  have essentially the same lifetimes, and hence  $\Delta m_{B,D} = 2|M_{12}|$ . Direct experimental measurements and limits on resultant mass differences are [12,15]:

$$\Delta m_K = (0.530 \pm 0.001) \times 10^{10} \text{ s}^{-1} = (3.49 \pm 0.006) \times 10^{-15} \text{ GeV} \quad (4.1)$$

$$\Delta m_{B_d} = (0.502 \pm 0.007) \times 10^{12} \text{ s}^{-1} = (3.36 \pm 0.04) \times 10^{-13} \text{ GeV} \quad (4.2)$$

$$\Delta m_{B_s} > 14.4 \times 10^{12} \text{ s}^{-1} = 0.99 \times 10^{-11} \text{ GeV} \quad (95 \% \text{ CL}) \quad (4.3)$$

$$\Delta m_D < 7 \times 10^{10} \text{ s}^{-1} = 0.5 \times 10^{-13} \text{ GeV} \quad (95 \% \text{ CL}). \quad (4.4)$$

The standard model accounts for the two measured mass differences,  $\Delta m_K$  and  $\Delta m_{B_d}$  and agrees with the limits on the other two mass differences  $\Delta m_{B_s}$  and  $\Delta m_D$  [12,14]. This thereby constrains non-SM contributions such as those from ETC gauge boson exchanges. For example, a recent SM fit gives  $\Delta m_{B_s} \simeq (18 - 21) \times 10^{12} \text{ sec}^{-1} = (1.2 - 1.4) \times 10^{-11} \text{ GeV}$  with uncertainties  $\sim \pm 3 \times 10^{12} \text{ sec}^{-1}$ , i.e.  $\sim 0.2 \times 10^{-11} \text{ GeV}$  [14]. Evidently, this is rather close to the current lower limit [12]. The standard model predicts that  $\Delta m_D \sim O(10^{-17}) \text{ GeV}$  [12], much smaller than its current experimental upper limit.

We denote the effective Hamiltonian density for the transition  $M^0 \leftrightarrow \bar{M}^0$ , where  $M^0 = q_j \bar{q}_k$ , as

$$\mathcal{H}_{eff} = \sum_{\chi, \chi'} c_{\bar{j}k;\chi\chi'} \mathcal{O}_{\bar{j}k;\chi\chi'} \quad (4.5)$$

with

$$\mathcal{O}_{\bar{j}k;\chi\chi'} = [\bar{q}_{j\chi} \gamma_\lambda q_{k\chi}] [\bar{q}_{j\chi'} \gamma^\lambda q_{k\chi'}] \quad (4.6)$$

where  $\chi$  and  $\chi'$  denote chirality and the  $c_{\bar{j}k;\chi,\chi'}$  are coefficients with dimensions of inverse mass squared. Summations over color and spinor indices are understood. We then have  $M_{12} = \langle M^0 | \int d^3x \mathcal{H}_{eff} | \bar{M}^0 \rangle$

While SM box diagrams contribute only to  $\mathcal{O}_{\bar{j}k;LL}$ , ETC gauge boson exchange can contribute to  $\mathcal{O}_{\bar{j}k;\chi\chi'}$  with  $\chi\chi' = LL, LR, RR$ . Indeed, if the ETC couplings to the quarks are vectorial, the ETC contribution to the effective Hamiltonian density for  $M^0 \leftrightarrow \bar{M}^0$  transitions has the simple form

$$\mathcal{H}_{eff,ETC} = c_{\bar{j}k} [\bar{q}_j \gamma_\lambda q_k] [\bar{q}_j \gamma^\lambda q_k] . \quad (4.7)$$

Other operators are induced by renormalization group running. We neglect these renormalization effects here, since they do not affect substantially our results. We calculate the ETC contributions to the coefficients of the above four-fermion operators at the scale  $\Lambda_{TC}$ , studying their dependence on the ETC-breaking scales and on the mixing angles in the quark mass matrices. To obtain physical predictions, we then sandwich the operator in Eq. (4.6) between  $M^0$  and  $\bar{M}^0$  states, taking account of the two different color contractions [16], and use, as input, estimates of the relevant hadronic matrix elements  $\langle M^0 | \mathcal{O}_{\bar{j}k;\chi\chi'} | \bar{M}^0 \rangle$ . For recent lattice measurements of these matrix elements, see, e.g., [17].

### A. $K^0 - \bar{K}^0$ Mixing

Let us start from the (CP-conserving) mass difference  $\Delta m_K = 2\text{Re}(M_{12})$ , assuming that the  $L$  and  $R$  components of the down-type quarks transform either according to the same (fundamental) ETC representation or according to relatively conjugate representations. The two cases are very different, hence we treat them separately.

#### 1. Conjugate Representations

To construct the amplitude  $s\bar{d} \rightarrow d\bar{s}$  for the case of conjugate representations, we need the  $s - d$  coupling to ETC vector bosons expressed in terms of fermion mass eigenstates given in Eqs. (3.19) and (3.20). The key point is that there is a contribution to the amplitude without the necessity of any ETC mixing [8]. This is due to the fact that with this assignment,  $d_L$  and  $s_L$  both have  $U(1)^3$  generational charges that are opposite to the charges of  $d_R$  and  $s_R$ , and hence  $[\bar{d}_L \gamma_\lambda s_L][\bar{d}_R \gamma_\lambda s_R]$  is invariant under the  $U(1)^3$  generational symmetry.

Specifically, an  $s_L \bar{d}_L$  pair in the initial-state  $\bar{K}^0$  can annihilate to produce a  $V_1^2$ . It can then directly create a  $d_R \bar{s}_R$  in the final-state  $K^0$  because the right-handed components transform according to the conjugate fundamental representation. Similarly, a  $s_R \bar{d}_R$  pair in the initial  $\bar{K}^0$  can produce a  $V_2^1$  that directly creates a  $d_L \bar{s}_L$  in the final  $K^0$ . Since ETC is strongly interacting at the relevant scale,  $\Lambda_1$ , the lowest-order ETC amplitude provides only a rough estimate, which is

$$c_{\bar{d}s;LR} \simeq \left( \frac{g_{ETC}}{\sqrt{2}} \right)^2 \frac{\zeta}{M_1^2} \simeq \frac{8\zeta}{\Lambda_1^2} \quad (4.8)$$

where  $\zeta$  is a phase factor of unit modulus, and we have used Eq. (3.1). While higher-order ETC contributions are important, it is not expected that they will substantially modify this estimate. Even with  $\Lambda_1$  as large as  $10^3$  TeV, as given in Eq. (3.2), the estimate (4.8) leads to a value of  $\Delta m_K$  that is nearly two orders of magnitude larger than the experimental value [8]. As emphasized already, we regard this as a serious problem for these models, despite their success at producing intra-generational mass splittings.

It is possible for terms due to mixing to cancel the above contribution so as to produce an acceptably small result, but this would require that these two terms have similar magnitudes. This could happen, since contributions due to fermion mixing which are nominally proportional to  $1/\Lambda_j^2$  with  $j = 1$  or  $j = 2$  involves mixing angle factors that are naturally small, so the actual size of such terms might be as small as  $1/\Lambda_1^2$ . However, we regard this as very unlikely, since there is no symmetry reason for it and the hadronic matrix elements are different; it would thus require fine-tuning.

## 2. Vectorial Representation

With both the  $L$  and  $R$  components of the down-type quarks in the fundamental ETC representation, all the four-fermion operators entering the  $K^0 - \bar{K}^0$  amplitude violate the global  $U(1)^3$  generational symmetry. The  $s\bar{d}$  quark pair in the initial  $\bar{K}^0$  has the generational quantum numbers (i.e., ETC group index structure) given by  $V_1^2$ , and this cannot directly (without ETC mixing) produce the  $d\bar{s}$  quarks in the final-state  $K^0$  with its ETC group index structure given by  $V_2^1$ . In order for this transition to proceed, ETC mixing is necessary. It must transform the initial state with the ETC generational index structure of  $V_1^2$  to the final state with the structure of  $V_2^1$ . This occurs via loops of SM-singlet fermions at ETC scales below  $\Lambda_1$  [8] and hence leads to strong suppression of the amplitude.

An example of the relevant ETC mixing is that among the  $SU(2)_{TC}$ -doublet ETC bosons, leading to the off-diagonal quark mass matrices. It is incorporated in the couplings (3.16). For this transition, we need the quantity  $(A_2^1)^\lambda$  appearing in the vertex  $\bar{d}\gamma_\lambda(A_2^1)^\lambda s$  in Eq. (3.16). We keep mixing terms involving couplings to the massive ETC vector bosons with the lowest two masses,  $\Lambda_3$  and  $\Lambda_2$ , and perform a Taylor series expansion in small rotation angles, truncating it after the most relevant terms:

$$\begin{aligned} (A^\lambda)_2^1 &= e^{i(\beta_1^{(d)} - \beta_2^{(d)})} \frac{(V^\lambda)_2^1}{\sqrt{2}} - e^{-i\delta^{(d)}} \theta_{13}^{(d)} \theta_{23}^{(d)} \frac{(V^\lambda)_{d3}}{\sqrt{3}} - \theta_{12}^{(d)} \frac{3(V^\lambda)_{d2}}{2\sqrt{6}} \\ &+ e^{i(\beta_3^{(d)} - \beta_2^{(d)} - \delta^{(d)})} \theta_{13}^{(d)} \frac{(V^\lambda)_2^3}{\sqrt{2}} + e^{i(\beta_2^{(d)} - \beta_3^{(d)})} \theta_{12}^{(d)} \theta_{23}^{(d)} \frac{(V^\lambda)_3^2}{\sqrt{2}}. \end{aligned} \quad (4.9)$$

Note that we have used the convention of Eq. (3.18) in which the  $\alpha$  phases have been rotated away.

Consider the exchange of an ETC gauge boson between the quarks, in both  $s$ - and  $t$ -channels. Wick-contracting the ETC gauge fields  $(A^\lambda)_2^1$ , keeping only the leading small-rotation-angle terms and setting the momentum in the ETC gauge boson propagator to zero, we have

$$c_{\bar{d}s} \mathcal{O}_{\bar{d}s} \simeq \left[ \frac{6}{\Lambda_2^2} (\theta_{12}^{(d)})^2 + \frac{16}{3\Lambda_3^2} e^{-2i\delta^{(d)}} (\theta_{13}^{(d)} \theta_{23}^{(d)})^2 \right] [\bar{d}\gamma_\lambda s][\bar{d}\gamma^\lambda s]. \quad (4.10)$$

In this and subsequent expressions, we use the relation in Eq. (3.1) to re-express  $g_{ETC}^2/M_j^2 \simeq 16/\Lambda_j^2$  for  $j = 1, 2, 3$ . Higher-order ETC contributions are important because of the strong-coupling nature of the ETC theory at this scale. They are incorporated in the coefficient of order unity that implicitly multiplies the right-hand side of Eq. (4.10). We insert the operator (4.10) between  $|\bar{K}^0\rangle$  and  $\langle K^0|$  states and perform the  $\int d^3x$  integral to obtain  $M_{12}$ .

Since the standard model can fit the experimental value of  $\Delta m_K$  up to the uncertainty due to long-distance QCD effects in the calculation of this quantity, we require that the ETC contribution to  $\Delta m_K$  be less than about 30 % of the SM contribution. Conservatively assuming no near cancellations involving the terms in Eq. (4.10), we obtain the following bounds:

$$|\theta_{12}^{(d)}| \lesssim 0.01, \quad (4.11)$$

$$|\theta_{13}^{(d)} \theta_{23}^{(d)}| \lesssim 0.4 \times 10^{-3} \quad (4.12)$$

Values of the angles satisfying the inequalities (4.11) and (4.12) are plausible; these angles are calculable, and in viable models are functions of ratios of smaller to larger ETC scales [6,8].

We note finally that early studies of  $K^0 - \bar{K}^0$  mixing in ETC models, although not based on UV-complete models, often took the ETC interactions to be vectorial. Interestingly, the studies of which we are aware failed to observe that there would naturally be suppression of the amplitude due to the necessity of mixing.

### 3. The $\epsilon_K$ Parameter

We turn next to the CP-violating effects, continuing to assign  $L$  and  $R$  components of down-type fields to the same ETC representation. We define the action of the CP operator as  $CP|K^0\rangle = e^{i\xi_K}|\bar{K}^0\rangle$ ,  $CP|\bar{K}^0\rangle = e^{-i\xi_K}|K^0\rangle$  on the neutral kaon, and  $CPq(CP)^{-1} = e^{i\xi_q}\gamma^0 C\bar{q}^T$  for a quark field  $q$ , where  $\xi_K$  and  $\xi_q$  are convention-dependent phases (e.g. [18]). The CP eigenstates in the neutral kaon system are given by the mixed states  $|K_{1,2}\rangle = (|K^0\rangle \pm e^{i\xi_K}|\bar{K}^0\rangle)/\sqrt{2}$ , with eigenvalues  $\pm 1$  respectively. They differ from the mass eigenstates by small CP-violating effects (indirect CP violation), so that the actual mass eigenstates are  $|K_S\rangle = (|K_1\rangle + \epsilon_K|K_2\rangle)/\sqrt{1+|\epsilon_K|^2}$  and  $|K_L\rangle = (|K_2\rangle + \epsilon_K|K_1\rangle)/\sqrt{1+|\epsilon_K|^2}$  with respective masses  $m_{K_L}$  ( $= m_{K_h}$ ) and  $m_{K_S}$  ( $= m_{K_\ell}$ ). Making use of approximations justified by experiment, namely  $|Im(\Gamma_{12})| \ll |Im(\Gamma_{12})|$ ,  $\Gamma_{K_S} \gg \Gamma_{K_L}$ , and  $\Delta m_K \simeq (1/2)\Gamma_{K_S}$ , one can derive a rephasing invariant expression for  $|\epsilon_K|$ ,

$$|\epsilon_K| \simeq \frac{|\text{Im}(M_{12} e^{i(\xi_K + \xi_d - \xi_s)} (V_{us}^* V_{ud})^2)|}{\sqrt{2} \Delta m_K |V_{us}^* V_{ud}|^2}. \quad (4.13)$$

In this expression, the convention-dependent  $\xi$  phases are removed by corresponding phases in  $M_{12}$ . Experimentally,  $|\epsilon_K| \simeq 2 \times 10^{-3}$ .

The ETC contributions to  $\text{Re}(M_{12})$  are small, so that  $\Delta m_K$  is determined mainly by the SM. We use the experimental value for  $\Delta m_K$  in the denominator of Eq. (4.13). In the numerator,  $M_{12}$  arises dominantly from the SM, with ETC making a smaller contributions. We focus on the latter.

The CKM factor  $V_{us}^* V_{ud}$ , defined according to Eq. (3.8), enters Eq. (4.13) because it multiplies the SM tree-level decay amplitude of  $K$  mesons into a final state of two pions with total isospin  $I = 0$ . We explicitly include it to denote the rephasing invariance of  $\epsilon_K$ . In the canonical parametrization of the CKM matrix,  $V_{ud}$  and  $V_{us}$  are real, but we use a more general form. We expand in small angles the CKM factor (3.8), in analogy to what we did in Eq. (4.10). Keeping terms up to quadratic order, we have

$$\begin{aligned} (V_{us}^* V_{ud})^2 &\simeq (\theta_{12}^{(d)})^2 - 2e^{-i(\beta_1^{(d)} - \beta_2^{(d)} - \beta_1^{(u)} + \beta_2^{(u)})} \theta_{12}^{(d)} \theta_{12}^{(u)} \\ &+ e^{-2i(\beta_1^{(d)} - \beta_2^{(d)} - \beta_1^{(u)} + \beta_2^{(u)})} (\theta_{12}^{(u)})^2. \end{aligned} \quad (4.14)$$

From the bound in (4.11), together with the approximate relation  $|V_{us}| \sim |\theta_{12}^{(u)} - \theta_{12}^{(d)}| \simeq 0.22$ , one can infer that  $|V_{us}| \simeq |\theta_{12}^{(u)}| \simeq 0.22 \gg \theta_{12}^{(d)}$ , so that, to a good approximation,  $(V_{us}^* V_{ud})$  is dominated by the term proportional to  $(\theta_{12}^{(u)})^2$ , and

$$\frac{(V_{us}^* V_{ud})^2}{|V_{us}^* V_{ud}|^2} \simeq e^{-2i(\beta_1^{(d)} - \beta_2^{(d)} - \beta_1^{(u)} + \beta_2^{(u)})}. \quad (4.15)$$

While mixing angles can naturally be small in ETC theories, arising as ratios of hierarchical ETC scales, there is no indication of a mechanism suppressing CP-violating phases. We take them to be  $\mathcal{O}(1)$ . To suppress the ETC contribution to  $\epsilon_K$ , the mixing angles must then be smaller than required to saturate (4.11) and (4.12), derived from  $\Delta m_K$ . From Eq. (4.13), requiring that the ETC contribution to  $\epsilon_K$  be smaller than 30% of the SM, we obtain the bounds

$$|\theta_{12}^{(d)}| \lesssim 10^{-3}, \quad (4.16)$$

$$|\theta_{13}^{(d)} \theta_{23}^{(d)}| \lesssim 10^{-4}. \quad (4.17)$$

These constraints can be plausibly satisfied in the class of ETC models analyzed here. We note that Eq. (4.16) indicates that the Cabibbo mixing angle arises dominantly from mixing in the up-quark sector. We will discuss the direct CP-violation in the kaon system and the associated quantity  $\epsilon'_K$  in Section V A 1.

The ETC mechanism for the natural suppression of flavor-changing effects (large scales together with small mixing angles) is rather different from the corresponding mechanism in the standard model. There, the  $K^0 - \bar{K}^0$  amplitude arises from box diagrams with two internal  $W$  lines in the  $s$  and  $t$  channels, and the small size of the imaginary part relative to the real part is explained as a consequence of the smallness of the charged-current couplings connecting the first- and second-generation quarks to the third-generation quarks. As in the case of ETC contributions, the smallness of the effect does not imply that the CP-violating phase itself is small.

## B. Other Mixings

### 1. $B_d - \bar{B}_d$ Mixing

The mixing amplitude  $M_{12}$  in the neutral  $B_d - \bar{B}_d$  system produces a mass difference  $\Delta m_{B_d}$ , and, via its CP violating complex phase, gives rise to CP-violation in the interference between mixing and decay amplitudes in  $B_d, \bar{B}_d$  decays. When two conjugate states  $B_d$  and  $\bar{B}_d$  decay to the same final state, the presence of state mixing between  $B_d$  and  $\bar{B}_d$  and the resultant time-dependent oscillations produce striking CP-asymmetries [19]. These have been measured at the asymmetric  $B$  factories Belle and BABAR. The cleanest mode  $B_d, \bar{B}_d \rightarrow J/\psi K_S$  yields, within the standard model, a precise measurement of the quantity  $\sin 2\beta \equiv \sin 2\phi_1$ . These experiments are in agreement with global SM fits. In contrast to the situation in the neutral kaon sector, this CP-violating effect is not small.

We analyze ETC contributions to this mixing where the  $L$  and  $R$  components of the down-type quarks transform according to the same, fundamental representation of the ETC group. In the interaction  $\bar{d}\gamma_\lambda (A^\lambda)_3^1 b$ , written in terms of quark mass eigenstates, the field  $(A^\lambda)_3^1$  is given by

$$\begin{aligned} (A^\lambda)_3^1 = & e^{i(\beta_1^{(d)} - \beta_3^{(d)})} \frac{(V^\lambda)_3^1}{\sqrt{2}} - e^{-i\delta^{(d)}} \theta_{13}^{(d)} \frac{(V^\lambda)_{d3}}{\sqrt{3}} \\ & + e^{i(\beta_2^{(d)} - \beta_3^{(d)})} \theta_{12}^{(d)} \frac{(V^\lambda)_3^2}{\sqrt{2}} - \theta_{12}^{(d)} \theta_{23}^{(d)} \frac{3(V^\lambda)_{d2}}{2\sqrt{6}} \\ & + e^{i(\beta_3^{(d)} - \beta_2^{(d)} - \delta^{(d)})} \theta_{13}^{(d)} \theta_{23}^{(d)} \frac{(V^\lambda)_2^3}{\sqrt{2}} \end{aligned} \quad (4.18)$$

where we have kept terms involving ETC vector bosons with masses  $\Lambda_3$  and  $\Lambda_2$ , and where we have expanded in small rotation angles. We focus on the  $\Lambda_3$  term, which should be representative of the total contribution.

As in  $K^0 - \bar{K}^0$  mixing, there is no contribution to the relevant four-fermion operators in the absence of mixing among the ETC gauge bosons. With this effect included, the ETC contribution to  $M_{12}$  is small compared to the SM contribution. Using Eq. (4.18), we estimate the part of this contribution arising from the ETC mixing that generates the off-diagonal terms in the quark mass matrix. In order for it not to upset the successful SM fit to the data (for  $\Delta m_{B_d}$  and the CP-violating asymmetry), and in light of the fact that in the heavy-quark meson systems the hadronic uncertainties are smaller than in the kaon system, we require that it be at most 0.1 compared with the measured absolute value of  $M_{12}$ . This yields the constraint

$$|\theta_{13}^{(d)}| \lesssim 1 \times 10^{-3}. \quad (4.19)$$

This constraint is plausibly satisfied in our ETC models for the same reason as given above: the rotation angles are naturally small, since they are calculable as ratios of smaller to larger ETC mass scales. The constraint (4.19) is comparable to the size of the actual CKM angle  $\theta_{13}$ .



## 2. $B_s - \bar{B}_s$ Mixing and $D^0 - \bar{D}^0$ Mixing

Continuing to take the  $L$  and  $R$  components of the down-type quarks to transform according to the same representation of the ETC group, there will be no ETC contribution to  $B_s - \bar{B}_s$  mixing without ETC mixing. Experimentally, given the lower bound on  $\Delta m_{B_s}$  in Eq. (4.4) [12], ETC mixing angles are not strongly constrained by the contributions to this process at the lowest scale  $\Lambda_3$ . As noted in Ref. [8], if relatively conjugate representations are employed for  $L$  and  $R$  components of down-type quarks, ETC contributions involving the exchange of a virtual  $V_2^3$  gauge boson will, even in the absence of mixing, render  $\Delta m_{B_s}$  considerably larger than the SM prediction.

The physics of  $D^0 - \bar{D}^0$  mixing is analogous to that of the systems already considered, but for the replacement with up-type quarks in the relevant operators. Again, if  $L$  and  $R$  components are assigned to relatively conjugate representations, then unacceptably large contributions arise from the exchange of ETC gauge bosons with no mixing required. If  $L$  and  $R$  components of the up-type quarks are in the same (fundamental) representation, then ETC mixing is required. Keeping mixing terms involving the two lightest ETC scales, and requiring that ETC contributions be smaller than the current upper limit (given in Eq. (4.4)), we obtain

$$|\theta_{12}^{(u)}| \lesssim 0.02, \quad (4.20)$$

$$|\theta_{13}^{(u)} \theta_{23}^{(u)}| \lesssim 10^{-3}. \quad (4.21)$$

Again, the ETC models of the class being considered, with vector-like ETC couplings, plausibly satisfy this bound.

We have now concluded that in the case of both the down-type quarks and the up-type quarks, their assignment to vector-like ETC representations can lead to an adequate suppression of  $M^0 - \bar{M}^0$  mixing. (We have used the fundamental representation, but the anti-fundamental would serve as well, as would other vector-like representations.) It is important to stress, however, that this cannot be done simultaneously, since it would lead to the absence of realistic intra-family mass splittings and CKM mixing. If the up-type quarks, for example, are assigned to a vector-like representation, say the fundamental, then the  $R$  components of the down-type quarks must be assigned to some other representation. We have shown that the choice of the anti-fundamental would lead to unacceptably large mixing, but one could consider other possibilities. The key criterion is that the relevant four-fermion operators violate the global  $U(1)^3$  symmetry. Thus, bounds such as in (4.21) and in (4.17) must not be considered together. At most one of them applies – to those quarks in a vectorial (fundamental or anti-fundamental) ETC representation.

### 3. Muonium-Antimuonium Conversion

There is an interesting analog in the leptonic sector to these meson systems: the muonium atom, a bound state of an anti-muon and an electron. The phenomenology of the muonium-antimuonium system is described by an effective Lagrangian similar to those of the neutral meson systems. This is the first encounter with leptons in this paper. All the processes considered here involving leptons, including the present one, will allow the charged leptons to be in any ETC representation. Suppose, for example, that the  $L$  and  $R$  components of charged leptons transform respectively according to the fundamental and conjugate-fundamental representations. Then the four-fermion operator  $[\bar{e}_L \gamma_\lambda \mu_L][\bar{e}_R \gamma^\lambda \mu_R]$  preserves the  $U(1)^3$  global generational symmetry inherited from ETC. It receives contributions at scale  $\Lambda_1$ , with no further suppression due to mixing. But even this makes the effect more than three orders of magnitude smaller than the experimental upper limit on the muonium-antimuonium amplitude [20]. Additional contributions to the relevant four-fermion operators from the lowest ETC scale are suppressed by mixing angles, and impose very mild bounds on these angles, which are easily satisfied. If the  $L$  and  $R$  components are assigned to ETC representations that do not lead to a  $U(1)^3$ -invariant four-fermion amplitude, then there is no non-mixing contribution, and the mixing contributions lead to mild bounds.

## V. OTHER PROCESSES

We next consider constraints on ETC mixing angles coming from other processes: rare  $K$  decays and leptonic transitions. In each case, the current phenomenological constraints can be satisfied with any ETC-representation assignment of the  $L$  and  $R$  components of the quarks and charged leptons. Even when the relevant four-fermion operators induced by ETC interactions preserve the global  $U(1)^3$  symmetry, so that no ETC mixing is required to produce them, the large scale  $\Lambda_1$  suppresses them below experimental observability. Yet, in some cases interesting constraints emerge by considering the additional contributions which involve the lowest ETC scale  $\Lambda_3$ , through ETC mixing. They can be satisfied naturally in the class of models being considered.

### A. $K$ Decays

Rare pseudoscalar meson decays have been extensively investigated experimentally, and represent an important test of any model of new physics. This is especially true for decays of the kaons. Other meson decays, i.e.  $B_d \rightarrow \phi K_S$ ,  $B_s \rightarrow \mu^+ \mu^-$ , do not yet provide significant new bounds [21].

1.  $K \rightarrow 2\pi$  and  $\epsilon'_K/\epsilon_K$

CP violation in the neutral kaon system has conventionally been classified as (i) indirect, occurring via the mixing of CP-even and CP-odd states to form the mass eigenstates, manifested in the complex parameter  $\epsilon_K$  defined in Eq. (4.13), and (ii) direct, arising from the interference between contributions containing different CP-violating phases to the decay amplitudes of a  $K$  meson into a final state with two pions, manifested in the parameter  $\epsilon'_K$ . Experimentally,  $\text{Re}(\epsilon'_K/\epsilon_K) = (1.8 \pm 0.4) \times 10^{-3}$  [12].

In the standard model, direct CP violation arises at the one loop level from penguin diagrams. There are uncertainties in theoretical estimates of  $\text{Re}(\epsilon'_K/\epsilon_K)$  owing to difficulties in calculating the relevant matrix elements and in choosing input values of some parameters such as the strange quark mass [22]. The experimentally measured value of  $\text{Re}(\epsilon'_K/\epsilon_K)$  is consistent with the SM prediction, to within these one-order-of-magnitude uncertainties. Hence we can deduce an indicative bound on ETC, by neglecting the SM contributions to  $\epsilon'_K$ , by estimating ETC contributions and by requiring that they do not exceed the experimental result.

We define  $A_{0,2} e^{i\delta_{0,2}} = \langle \pi\pi_{(0,2)} | H_{eff} | K^0 \rangle$ , explicitly factoring out the (CP-conserving) strong phases  $\delta_{0,2}$  due to final-state interactions. (The standard model produces the operator  $[\bar{s}_L \gamma_\lambda u_L][\bar{u}_L \gamma^\lambda d_L]$  via tree-level exchange of a W boson.) All the relevant four-fermion operators contain one s-quark and three first-family quarks. Thus all the operators that arise from ETC interactions, independent of the ETC-representation assignment of the  $L$  and  $R$  components, violate the  $U(1)^3$  global symmetry and require ETC mixing in order to be generated. Consider, for example, the operator  $[\bar{s}_L \gamma_\lambda d_L][\bar{u}_L \gamma^\lambda u_L]$ . We estimate (using, for QCD effects, the phenomenological approach reviewed in [22], in which the experimental result  $|A_2/A_0| \simeq 0.05$  is built in) that its contribution to  $A_I$  is of order  $0.01 A_I^{SM} \theta_{13}^{(d)L} \theta_{23}^{(d)L} (\theta_{13}^{(u)L})^2 \omega / (V_{us}^* V_{ud})$ , where  $\omega$  is an  $\mathcal{O}(1)$  phase factor containing the phases in Eqs. (3.9)-(3.13), and where the numerical factor 0.01 includes the ETC scale  $\Lambda_3$  as well as QCD effects. Using the approximate relation  $|\text{Re}(\epsilon'_K/\epsilon_K)| \simeq |\text{Im}(A_2/A_0)|/|\sqrt{2}\epsilon_K|$ , and the experimental value of  $|\epsilon_K|$ , we obtain the mild constraint  $(\theta_{13}^{(d)L} \theta_{23}^{(d)L})^{1/2} |\theta_{13}^{(u)L}| \lesssim 0.04$  with similar bounds on other combinations of mixing angles. These bounds are not particularly restrictive, compared with those from  $\Delta m_K$  and  $\epsilon_K$ .

This limit on mixing angles is milder than those in (4.16)-(4.17), derived from  $\epsilon_K$ , which is bigger experimentally. Both quantities can be estimated as the ratio of the CP-violating contribution to an amplitude over its CP-conserving part (the mixing amplitude  $M_{12}$  for  $\epsilon_K$ , the decay amplitudes  $A_{0,2}$  for  $\epsilon'_K$ ). The dominant CP-conserving SM contribution to  $A_0$  arises at tree level, with a very modest CKM suppression, so that new physics contributions arising at scales much larger than the electroweak are relatively strongly suppressed. By contrast,  $M_{12}$  arises in the standard model from strongly GIM-suppressed loop diagrams, and hence is much more sensitive to new physics at high scales, if no analog to the GIM suppression is present.

## 2. $K^+ \rightarrow \pi^+ \mu^\pm e^\mp$

Current limits are  $BR(K^+ \rightarrow \pi^+ \mu^+ e^-) < 2.8 \times 10^{-11}$  and  $BR(K^+ \rightarrow \pi^+ e^+ \mu^-) < 5.2 \times 10^{-10}$  from the E865 experiment at BNL [23]. The  $K^+ \rightarrow \pi^+ \mu^+ e^-$  decay arises from the elementary process  $\bar{s} \rightarrow \bar{d} \mu^+ e^-$  with a spectator  $u$ . Because both the  $L$  and  $R$  components of the down-type quarks and leptons enter the amplitudes for this process, there will typically be an amplitude invariant under the  $U(1)^3$  generational symmetry. Only in the case that the  $L$  and  $R$  components of the quarks are assigned to the same ETC-representation, and the  $L$  and  $R$  components of the charged leptons are also assigned to the same representation (but conjugate to that of the down-type quarks), will this not be the case. Excluding this possibility to focus on the worst case, the amplitude can occur with no ETC mixing, and is of order  $1/\Lambda_1^2$ . But even this is sufficiently small relative to the above experimental limit. Since this contribution, without ETC mixing, is well below the experimental bound, one can anticipate that contributions involving mixing will not lead to especially tight constraints on the mixing angles. An estimate as in Section IV, expanding in small rotation angles, leads to a bound much milder than those derived so far.

The situation with  $K^+ \rightarrow \pi^+ \mu^- e^+$  is much the same. Except for one possible choice of ETC-representation assignments, there will be a  $U(1)^3$ -invariant amplitude, with no ETC mixing required, and with coefficient  $1/\Lambda_1^2$ . Again, this contribution is below the experimental bound, and the additional contribution involving mixing leads to only a mild bound on mixing angles.

## 3. $K_L \rightarrow \mu^\pm e^\mp$

The current limit on the branching ratio is  $BR(K_L \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-12}$  from the E871 experiment at BNL [24]. The peculiarity of this process is that the hadronic matrix element arises only from the axial-vector part of the relevant bilinear quark operator.

In the case where ETC interactions of the quarks are not vectorial, there is always a contribution to these decays at scale  $\Lambda_1$ , with no mixing required, because it involves a second generation fermion (or antifermion) in both the initial and final state. Thus one of the possible four-fermion operators preserves the  $U(1)^3$  global symmetry. Given the value of  $\Lambda_1$  in Eq. (3.2), we estimate this to lead to a branching ratio  $BR(K_L \rightarrow \mu^\pm e^\mp) \approx 10^{-12}$ , still allowed by current experimental bounds, but potentially observable in next-generation experiments. If the ETC coupling to the down-type quarks is vectorial, this process cannot be generated to any order in ETC interactions, even if the relevant four-fermion operator does not violate  $U(1)^3$ . Nonzero contributions depending on ETC interactions arise from graphs involving both electroweak gauge bosons and ETC exchange. These are expected to yield an amplitude of order  $(\alpha/\pi)(1/\Lambda_1^2)$  and hence a rate that is safely smaller than the above experimental limit.

#### 4. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Here we consider the decay  $K^+ \rightarrow \pi^+$  + missing (weakly interacting) neutrals. The branching ratio for this decay has been measured by the E787 and E949 experiments at BNL, with the result [25]  $BR(K^+ \rightarrow \pi^+ + \text{missing neutrals}) = (1.57^{+1.75}_{-0.82}) \times 10^{-10}$ . In the standard model the rate for this observed decay is the (incoherent) sum of the rates for each of the three individual decays  $K^+ \rightarrow \pi^+ \nu_\ell \bar{\nu}_\ell$ , where  $\nu_\ell$  are the three light neutrinos. These decays arise at the one-loop level. The SM prediction for the branching ratio is  $\sum_\ell BR(K^+ \rightarrow \pi^+ \nu_\ell \bar{\nu}_\ell) = (0.77 \pm 0.11) \times 10^{-10}$  [26], consistent with the measurement. In this process and in others involving neutrino final states, we neglect neutrino masses, since they are very small compared to the other masses in the process.

Whether the  $L$  and  $R$  components of the down-type quarks are assigned to the same or conjugate ETC representations, there will be a contribution to the underlying process  $\bar{s} \rightarrow \bar{d} + \nu_e \bar{\nu}_\mu$  with no ETC mixing and proportional to  $1/\Lambda_1^2$ . It can proceed, for example, by the exchange of a single  $V_2^1$  ETC gauge boson. With  $\Lambda_1 \approx 10^3$  TeV, the ETC contribution is comfortably below the experimental bound.

Other contributions arise from ETC mixing, for example those generating off-diagonal quark mass terms. The process can proceed by  $\bar{s}$  emitting a virtual  $V_{d3}$  ETC gauge boson, going to a  $\bar{d}$ . The  $V_{d3}$  can then produce, with no further ETC mixing required, the pair of interaction eigenstates  $\nu_\tau \bar{\nu}_\tau$ . Other contributions include the exchange of a  $V_{d2}$  ETC gauge boson, but do not impose significant bounds. Higher-order contributions are understood to be present, owing to the strong-coupling nature of the ETC theory, but these have the same overall generational index structure as the lowest-order exchanges. Requiring the contribution induced by  $V_{d3}$  exchange be less than the SM prediction for the branching ratio, we obtain the limit

$$|\theta_{13}^{(d)\chi} \theta_{23}^{(d)\chi'}| \lesssim 10^{-3}, \quad (5.1)$$

where we retain the labels  $\chi, \chi' = L, R$ , because the bound does not depend on the chirality assignments.

These bounds are somewhat weaker than the bound we derived earlier from the measurement of  $\epsilon_K$  in Eq. (4.17). Since plausible values of the above angles suggest that the left-hand side of eq. (5.1) could nearly saturate the limit, it is possible that ETC contributions to this decay could amount to a significant fraction of the SM branching ratio.

#### 5. $K_L \rightarrow \pi^0 \nu \bar{\nu}$

This is, experimentally, the decay  $K_L \rightarrow \pi^0$  + missing weakly interacting neutrals. In the standard model, it is  $K_L \rightarrow \sum_\ell \pi^0 \nu_\ell \bar{\nu}_\ell$ . This is of interest because, although it is not manifestly a CP-violating decay, the main contribution in the standard model turns out to involve a direct CP-violating amplitude [27]. As with  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , the amplitude can be calculated accurately in the standard model terms of the CKM

mixing parameters. From a current global fit, one obtains the SM prediction  $BR(K_L \rightarrow \sum_\ell \pi^0 \nu_\ell \bar{\nu}_\ell) = (2.6 \pm 0.5) \times 10^{-12}$  [26]. The current experimental limit,  $BR(K_L \rightarrow \sum_\ell \pi^0 \nu_\ell \bar{\nu}_\ell) < 5.9 \times 10^{-7}$  [12], is not nearly sensitive to the SM prediction, but the future KOPIO experiment at BNL plans to measure the decay and test this prediction. The ETC contribution to the decay  $K_L \rightarrow \pi^0 +$  missing weakly interacting neutrals arise in a manner similar to that for  $K^+ \rightarrow \pi^+ +$  missing neutrals and might ultimately produce a measurable deviation relative to the SM prediction.

## B. Leptonic Processes

ETC-boson exchanges analogous to those discussed for quarks induce also four-fermion operators of relevance for experimentally accessible leptonic processes. They lead to only mild constraints, and, as with the semi-leptonic processes above, allow the  $L$  and  $R$  components of the charged leptons to be in any representation of the ETC group.

### 1. $\mu^+ \rightarrow e^+ e^+ e^-$

Experimentally,  $BR(\mu^+ \rightarrow e^+ e^+ e^-) < 1.0 \times 10^{-12}$ . Since this process involves only a single second-generation fermion, the four-fermion amplitudes contributing to it necessarily violate the  $U(1)^3$  global symmetry. Thus ETC mixing is required whatever the ETC-representation assignments of the  $L$  and  $R$  components of the charged leptons. Proceeding as in Section IV, keeping terms involving the exchange of ETC vector bosons of the lowest two masses,  $\Lambda_3$  and  $\Lambda_2$ , and performing an expansion in small rotation angles, we have the following ETC contribution to the effective Hamiltonian for  $\mu^- \rightarrow e^- e^- e^+$ :

$$\left[ \frac{16}{3\Lambda_3^2} e^{-i\delta^{(\ell)}} (\theta_{13}^{(\ell)})^3 \theta_{23}^{(\ell)} + \frac{6}{\Lambda_2^2} (\theta_{12}^{(\ell)})^3 \right] [\bar{e} \gamma_\lambda \mu] [\bar{e} \gamma^\lambda e], \quad (5.2)$$

where we have dropped chirality labels on the mixing angles for simplicity. From the experimental upper limit on  $\mu^+ \rightarrow e^+ e^+ e^-$ , we get the bound  $|\theta_{13}^{(\ell)}|^{3/2} |\theta_{23}^{(\ell)}|^{1/2} \lesssim 0.006$ . This is a relatively mild bound, and easily accommodated in our models where mixing angles are ratios of hierarchical ETC scales.

### 2. $\mu \rightarrow e$ Conversion

A number of searches have been carried out for  $\mu \rightarrow e$  conversion in the Coulomb field of a nucleus. One contribution to  $\mu \rightarrow e$  conversion is from a process in which  $\mu \rightarrow e$  plus a virtual photon, which is exchanged with the nucleus. In ETC theories this process arises from the same type of amplitude that produces  $\mu \rightarrow e \gamma$ , which we have bounded earlier in Ref. [9]. We focus here on an additional ETC contribution in which, via mixing, the  $\mu$  makes a transition to an  $e$  with the exchange of a virtual  $V_{d3}$  with the nucleons. This process, involving only a single second-generation fermion, violates the  $U(1)^3$  symmetry, and hence requires ETC-mixing to be generated.

One can write the effective Hamiltonian for the  $\mu \rightarrow e$  conversion process as

$$\mathcal{H}_{\mu e} = \frac{G_F}{\sqrt{2}} \sum_{i=0,1} \left[ \bar{e} \gamma_\lambda (g_{VV}^{(i)} - g_{AV}^{(i)} \gamma_5) \mu J_{V,nuc.}^\lambda + \bar{e} \gamma_\lambda (g_{VA}^{(i)} - g_{AA}^{(i)} \gamma_5) \mu J_{A,nuc.}^\lambda \right] \quad (5.3)$$

where  $i = 0, 1$  refer to isoscalar and isovector contributions and  $J_{V,nuc.}$  and  $J_{A,nuc.}$  denote the effective nuclear vector and axial vector currents. Experimental bounds imply limits such as  $g_{VV}^{(0)} < 4 \times 10^{-7}$  [28]. From these we derive the bound  $[\theta_{13}^{(\ell)} \theta_{23}^{(\ell)}]^{1/2} |\theta_{13}^{(f)}| \lesssim 0.006$ , comparable to that from  $\mu^+ \rightarrow e^+ e^+ e^-$ , with the difference that it involves both charged lepton and quark mixing angles. We have again dropped chirality labels on the mixing angles for simplicity. The future MECO experiment at BNL projects a large improvement in sensitivity in the search for  $\mu \rightarrow e$  conversion [29].

### 3. Ordinary $\mu$ Decay

The exchange of virtual ETC gauge bosons adds new contributions to ordinary  $\mu$  decay, thus modifying the effective Fermi coupling with respect to the standard model. Although  $\mu^+$  decay is conventionally regarded as being  $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$ , as predicted by the standard model, at an experimental level it is simply  $\mu^+ \rightarrow e^+$  + unobserved weakly interacting neutrals. On the other hand, since the corrections to the SM decay rate we are considering are very small, the most important ETC contributions are the coherent ones (to the same final states as the standard model), so that we focus our attention on the operator  $[\bar{\mu}_L \gamma_\lambda e_L][\bar{\nu}_{eL} \gamma^\lambda \nu_{\mu L}]$ , which receives contributions both from  $W$  boson exchange and from ETC exchange.

This is another process in which ETC can contribute without any mixing required, since the relevant operator preserves the  $U(1)^3$  global symmetry. The SM coefficient is  $G_F/\sqrt{2} = g^2/(8m_W^2)$ . ETC exchange at scale  $\Lambda_1$  gives a contribution  $\simeq 8/\Lambda_1^2 \simeq 10^{-6} G_F/\sqrt{2}$ , and hence is negligible. There are also terms due to lepton mixing arising from much lower scales, such as the one involving the exchange of a virtual  $V_{d3}$ . In order not to modify substantially electroweak precision observables, we require ETC to contribute at most 0.1 % of the SM amplitude, and derive a weak bound compared to those of previous subsections, which is easily satisfied in our ETC models,

## VI. DISCUSSION AND CONCLUSIONS

Constraints from neutral flavor-changing processes were among the first concerns in studies of extended technicolor. In this paper we have reconsidered these constraints, focusing on the relevant four-fermion operators. We have taken account of the multi-scale nature of the ETC gauge symmetry breaking, in a class of ultra-violet complete models in which the TC theory is approximately conformal ("walking"), and the ETC gauge group commutes with the SM gauge group. Features such as intra-family mass splitting and CKM mixing are generated by the dependence of the ETC representation assignments of the SM fermions

on their assignments under the standard model. This work extends our earlier general study in Ref. [8] and specific studies of (dimension-five) dipole moment operators in Refs. [9] and [10].

We have described an approximate global generational  $U(1)^3$  symmetry, inherited by the low-energy effective theory from the underlying ETC gauge symmetry, that controls the coefficients of the four-fermion operators. Operators that violate this symmetry are suppressed not only by large ETC scales, but also by (small) mixing effects among ETC gauge bosons. Employing this symmetry classification, we have considered two, relatively simple types of assignments: those in which  $L$  and  $R$  components of quark (and techniquark) fields of a given charge transform according to the same (fundamental or anti-fundamental) representation of the ETC group, and those in which they transform according to the opposite (fundamental and anti-fundamental) ETC representations. Corresponding assignments for the charged leptons must also be made, but the choice is not critical for the phenomenology of this paper.

We have analyzed  $K^0 - \bar{K}^0$  and  $B_d - \bar{B}_d$  mixings, and limits on  $B_s - \bar{B}_s$  and  $D^0 - \bar{D}^0$  mixing. We have also considered the decays  $K^+ \rightarrow \pi^+ \mu^\pm e^\mp$ ,  $K_L \rightarrow \mu^\pm e^\mp$ ,  $\mu^+ \rightarrow e^+ e^+ e^-$ ,  $\mu \rightarrow e$  conversion in the field of a nucleus, and effects on  $\mu$  decay. ETC contributions are suppressed by the heaviness of the ETC scales involved, and for operators that violate the  $U(1)^3$  by the smallness of the requisite ETC mixing effects.

For the case in which  $L$  and  $R$  components of quarks of a given electric charge transform according to relatively conjugate representations, some dangerous four-fermion operators involving SM fields preserve the global  $U(1)^3$  symmetry, and hence are suppressed only by the ETC scales, with no mixing required. For example, in the case of down-type quarks this leads to a very large contribution to the  $K^0 - \bar{K}^0$  mixing amplitude, excluding the viability of such an assignment for  $s$  and  $d$  quarks. We note, however, that this assignment for the down-type quarks, with up-type quarks coupling vectorially to ETC, produces charged-current (CKM) flavor mixing, together with substantial intra-generational mass splitting such as  $m_t \gg m_b$ . The latter is achieved without having introduced new sources of custodial-SU(2) violation below the (large) ETC scales, and hence suppressing new physics contributions to the  $\rho$  parameter in precision electroweak physics.

For the other case, in which  $L$  and  $R$  components transform according to the same (fundamental or anti-fundamental) representation, no excessively large contributions to any of the processes we have considered are generated provided the ETC scales are large enough and ETC mixing effects are small enough. This is because the global  $U(1)^3$  symmetry forbids the most dangerous four-fermion operators, which can then arise only through ETC-mixing. This fact seems not to have been noticed in earlier studies of ETC theories. We have focused on those ETC mixings leading to the off-diagonal structure of the quark mass matrices, i.e. on the mixing angles parameterizing the unitary matrices that diagonalize these matrices. Interesting bounds on these mixing angles emerge in the up- and down- sectors separately, and thus constrain even mixing parameters that do not enter in the CKM matrix.

But we stress that one cannot simultaneously assign both the down-type quarks and the up-type quarks, to vector-like ETC representations, since it would, without additional ingredients, lead to the absence of



realistic intra-family mass splittings and CKM mixing. If the up-type quarks, for example, are assigned to a vector-like representation, say the fundamental, then the  $R$  components of the down-type quarks must be assigned to some other representation. We have shown that the choice of the anti-fundamental would lead to unacceptably large  $M^0 - \bar{M}^0$  mixing, but one could consider other possibilities. The key criteria are that the ETC scales be large enough while still generating realistic fermion masses (as in the present paper), *and* that the four-fermion operators describing  $M^0 - \bar{M}^0$  mixing violate the global  $U(1)^3$  symmetry. Thus, bounds such as in (4.21) and in (4.17) must not be considered together. At most one of them applies – to those quarks in a vectorial (fundamental or anti-fundamental) ETC representation. The bounds on their mixing angles constrain future ETC model-building, but can naturally be satisfied in the class of models considered here. Furthermore, values of fermion mixing angles consistent with our constraints still allow substantial deviations from the SM predictions, for both CP-conserving quantities such as  $\Delta m_K$ ,  $K^+ \rightarrow \pi^+ \mu^+ e^-$ ,  $K^+ \rightarrow \sum_\ell \pi^+ \nu \bar{\nu}$ , and similar mixings and decays, and for CP-violating quantities.

This suggests a direction for future ETC model building within the class considered here. One should try, in effect, to capture the best features of the above representation choices. The  $L$  and  $R$  components of quarks should be assigned to representations of the ETC gauge group in such a way as to suppress the down-type quark mass matrix relative to the up-type mass matrix, at least those parts that determine the b-quark mass relative to the t-quark. The assignments must also suppress dangerous four-fermion operators, specifically those describing  $M^0 - \bar{M}^0$  mixing, that is, they must render the relevant four-fermion operators non-invariant under the  $U(1)^3$  symmetry. We are currently exploring this possibility.

Ultraviolet-complete ETC theories take on a very ambitious task, to explain dynamically, with a very small number of parameters, electroweak symmetry breaking, fermion generations, masses, and mixing. It is not surprising that it is difficult to find an entirely successful model, but we believe that there is strong motivation to continue the search. The constraints that we have obtained in our previous papers and in the present analysis may provide some helpful guidance for this model-building enterprise.

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- [1] S. Dimopoulos, L. Susskind, Nucl. Phys. **B155**, 23, (1979); E. Eichten, K. Lane, Phys. Lett. B **90**, 125 (1980).
  - [2] B. Holdom, Phys. Lett. B **150**, 301 (1985); K Yamawaki, M. Bando, K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Appelquist, D. Karabali, L.C.R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986); T. Appelquist and L.C.R. Wijewardhana, Phys. Rev. D **35**, 774 (1987); Phys. Rev. D **36**, 568 (1987).

- [3] Our use of  $SU(2)_{TC}$  with one SM family of technifermions ( $N_f = 8$  technidoublets) is based on the inference from the two-loop beta function and gap equations that this theory confines, spontaneously breaks chiral symmetry, and has an approximate infrared-stable fixed point that gives rise to walking behavior. Efforts continue to try to use lattice gauge theory methods to answer the question, in a (vectorial)  $SU(N)$  gauge theory, of what the critical value is for the number of massless fermions in the fundamental representation at which the theory goes over from a phase with confinement and spontaneous chiral symmetry breaking to a nonabelian Coulomb phase; see, e.g., R. Mawhinney, Nucl. Phys. Proc. Suppl. **83**, 57 (2000) and Y. Iwasaki et al., Phys. Rev. D **69**, 014507 (2004).
- [4] T. Appelquist and J. Terning, Phys. Lett. **B315**, 139 (1993); T. Appelquist, J. Terning, L.C.R. Wijewardhana, Phys. Rev. Lett. **77**, 1214 (1996); *ibid.* **79**, 2767 (1997).
- [5] T. Appelquist, J. Terning, Phys. Rev. D **50**, 2116 (1994).
- [6] T. Appelquist and R. Shrock, Phys. Lett. **B548**, 204 (2002).
- [7] T. Appelquist and R. Shrock, Phys. Rev. Lett. **90**, 201801 (2003).
- [8] T. Appelquist, M. Piai and R. Shrock, Phys. Rev. D **69**, 015002 (2004) [hep-ph/0308061].
- [9] T. Appelquist, M. Piai and R. Shrock, Phys. Lett. B **593**, 175 (2004) [hep-ph/0401114].
- [10] T. Appelquist, M. Piai and R. Shrock, Phys. Lett. B **595**, 442 (2004) [hep-ph/0406032].
- [11] P. Sikivie, L. Susskind, M. B. Voloshin and V. I. Zakharov, Nucl. Phys. B **173**, 189 (1980).
- [12] See <http://pdg.lbl.gov>.
- [13] For recent discussions of CP violation in the ETC context, see K. Lane, hep-ph/0202255; A. Martin and K. Lane, hep-ph/0404107.
- [14] F. Parodi, in *Proc. Fifth International Conference on Hyperons, Charm, and Beauty Hadrons*, ed. C. Kalman et al., Nucl. Phys. B (Proc. Suppl.) **115** (2003) 212; M. Battaglia et al., hep-ph/0304132; M. Ciuchini et al., hep-ph/0307195; <http://cern.ch/ckm-workshop>; M. Bona et al., hep-ph/0408079; Y. Sakai (Belle) and M. Giorgi (BABAR), talks at the XXVI Int. Conf. on High Energy Physics, Beijing, available at <http://icp04.ihep.ac.cn>.
- [15] We use the updated values for  $B_d$  and  $B_s$  mixing from the Heavy Flavor Averaging Group, <http://www.slac.stanford.edu/xorg/hfag/> which have been incorporated in the 2004 PDG [12].
- [16] R. Shrock and S. Treiman, Phys. Rev. D **19**, 2148 (1979).
- [17] For recent reviews, see N. Yamada, in *Lattice 2002 - Proceedings of the XXth International Symposium on Lattice Field Theory*, eds. R. Edwards, et al., Nucl. Phys. B (Proc. Suppl.) **119**, 93 (2003); D. Becirevic in *Lattice 2003 - Proceedings of the XXI International Symposium on Lattice Field Theory*, ed. S. Aoki et al., Nucl. Phys. B Proc. Suppl. **129-130**, 34 (2004).
- [18] See, e.g., G. Branco, L. Lavoura, and J. Silva, *CP Violation* (Oxford Univ. Press, Oxford, 1999); I. Bigi and A. I. Sanda, *CP Violation* (Cambridge Univ. Press, Cambridge, 2000).
- [19] A. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981).
- [20] Y. Kuno and Y. Okada, Rev. Mod. Phys. **73**, 151 (2001) [hep-ph/9909265]; L. Willmann *et al.*, Phys. Rev. Lett. **82**, 49 (1999) [hep-ex/9807011].
- [21] Determinations of  $\sin 2\beta$  from  $b \rightarrow sc\bar{c}$  decay modes such as  $B_d^0, \bar{B}_d^0 \rightarrow J/\psi K_S$  yield  $\sin 2\beta = 0.726 \pm 0.037$  while determinations from  $b \rightarrow sq\bar{q}$  penguin modes, such as  $B_d \rightarrow \phi K_S, B_d \rightarrow (K\bar{K})_{nonres.} K_S$  and  $B_d \rightarrow \eta' K_S$ , yield  $\sin 2\beta = 0.42 \pm 0.10$  (BABAR) and  $\sin 2\beta = 0.43^{+12}_{-0.11}$  (Belle). The latter values differ by about  $2.5\sigma$  from the  $b \rightarrow sc\bar{c}$  value of  $\sin 2\beta$ . New physics affecting the decay amplitudes of these processes, in particular in the class of ETC theories we are discussing [10], could play a role in explaining this difference.
- [22] See, e.g., S. Bertolini, M. Fabbrichesi and J. O. Eeg, Rev. Mod. Phys. **72**, 65 (2000).
- [23] R. Appel et al., Phys. Rev. Lett. **85**, 2450 (2000). Analysis of further BNL E865 data reduces slightly the upper limit on  $K^+ \rightarrow \pi^+ \mu^+ e^-$  (M. Zeller, private communication; A. Sher thesis; E865 Collab. to be published.)
- [24] D. Ambrose et al., Phys. Rev. Lett. **81**, 5734 (1998).
- [25] S. Adler et al., Phys. Rev. Lett. **79**, 2204 (1997); *ibid* **84**, 3768 (2000); *ibid* **88**, 041803 (2002); hep-ex/0403034; V. Anisimovsky et al., hep-ex/0403036.
- [26] See, e.g., G. Isidori, hep-ph/0307014.
- [27] L. Littenberg, Phys. Rev. D **39**, 3322 (1989); hep-ex/0201026, hep-ex/0212005; <http://www.bnl.gov/rsvp/KOPIO.htm>.
- [28] C. Dohmen et al., Phys. Lett. B **317**, 631 (1993); W. Honecker et al., Phys. Rev. Lett. **76**, 200 (1996); A. van der Schaaf, Nucl. Instr. Meth. A **503**, 281 (2003).
- [29] See <http://mecop.ps.uci.edu>.